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LETTER TO THE EDITORS

Comments on "Foundation of the interfacial area transport equation and its closure relations"

In a recent paper by Kocamustafaogullari and Ishii (hereafter—KI) [1], the following fluid particle number density transport equation is used

$$\frac{\partial f}{\partial t}+\nabla\cdot(f\mathbf{v}_p)=S_{cb}+S_{ph}\equiv S. \tag{1}$$

Here $f(\mathbf{x}, v, t)$ is the particle density distribution function by their coordinates and volumes v, $\mathbf{v}_p(\mathbf{x}, v, t)$ is the particle velocity, S_{cb} is the fluid particle source (or sink) rate due to coalescence and breakup, and S_{ph}—due to phase change. While deriving equation (1), KI applied 'a simple procedure accounting for the fluid particles entering and leaving a control volume through different mechanisms...'. However, they considered only the geometric space, instead of the phase space accounting for all parameters of particle state. As a result, KI lost one term in the l.h.s. of equation (1).

Let $\Xi = X\times V$ be a four-dimensional phase space consisting of vectors $\xi = [\mathbf{x}, v]$, and $d\xi/dt = \mathbf{u}$ be the equation of dynamic trajectories in the space Ξ (obviously, $u_k = v_{pk}$, $\kappa = 1, 2, 3$; $u_4 = dv/dt \equiv q$). For a fixed volume τ in the space Ξ which is limited by surface F, we can write down

$$\frac{d}{dt}\int_\tau f\,d\xi = -\oint_F f\mathbf{u}\cdot d\mathbf{F}+\int_\tau S\,d\xi. \tag{2}$$

Supposing that the function f is sufficiently regular and using the Gauss theorem, instead of equation (2) we have

$$\int_\tau\left(\frac{\partial f}{\partial t}-\nabla_\xi\cdot(f\mathbf{u})-S\right)d\xi=0$$

or, taking into account that the volume τ is arbitrary,

$$\frac{\partial f}{\partial t}+\sum_{k=1}^{4}\frac{\partial(fu_k)}{\partial\xi_k}=S \tag{3}$$

(the vector ∇_ξ has the components $\partial/\partial\xi_k$, $k=\overline{1,4}$). Equation (3) can be rewritten in the form

$$\frac{\partial f}{\partial t}+\nabla\cdot(f\mathbf{v}_p)+\frac{\partial}{\partial v}(fq)=S. \tag{4}$$

Comparing equations (1) and (4) we see that KI lost the third term in the l.h.s. *A propos*, the form of the transport equation like (4) is generally accepted [2–4].

This error further appears in KI's equations for the total number of particle, interfacial area concentration and void fraction (see equations (8), (12) and (24) in paper [1]). For example, in the l.h.s. of equation (8) in paper [1] the term $f(v_{max})q(v_{max})-f(v_{min})q(v_{min})$ is lost.

It should be noted, in addition, that KI calculated the S_{cb} term in equation (1) only for the particular case of particle full coalescence at collisions. At the same time, it is known that in many practical situations, droplet collisions lead not only to coalescence but to breakup as well. A general theory of this phenomenon, and equations for S_{cb} in the general case can be found in ref [4].

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